Computer Graphics - Excercise 1

1.1.1

a)

- Matrix Multiplication $\widehat{=}$ transformation
- Inverse of a transformation $\tilde{T}^{-1}$

$$\begin{aligned}
\Rightarrow \qquad & \tilde{T} * \tilde{T}^{-1} = \tilde{T}^{-1} * \tilde{T} = \tilde{E} && \mid *\tilde{p} \\
& \tilde{T}^{-1} * \tilde{T} * \tilde{p} = \tilde{E} * \tilde{p} && \Big| \, \tilde{T} * \tilde{p} = \tilde{p'} \\
\Rightarrow \qquad & \tilde{E} * \tilde{p} = \tilde{p}, \quad \tilde{T}^{-1} * \tilde{p'} = \tilde{p}
\end{aligned}$$

b)

For $R_1 \times R_2$

$$\begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} e & f & 0 \\ g & h & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a \times e + b \times g & a \times f + b \times h & 0 \\ c \times e + d \times g & c \times f + d \times h & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

For $R_2 \times R_1$

$$\begin{pmatrix} e & f & 0 \\ g & h & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a \times e + c \times f & b \times e + d \times f & 0 \\ a \times g + c \times h & b \times g + d \times h & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow R_1 \times R_2 \neq R_2 \times R_1 \Rightarrow$ does not commute

For $T_1 \times T_2$

$$\begin{pmatrix}1 & 0 & a\\0 & 1 & b\\0 & 0 & 1\end{pmatrix} \times \begin{pmatrix}1 & 0 & c\\0 & 1 & d\\0 & 0 & 1\end{pmatrix} = \begin{pmatrix}1 & 0 & a+c\\0 & 1 & b+d\\0 & 0 & 1\end{pmatrix}$$

For $T_2 \times T_1$

$$\begin{pmatrix}1 & 0 & c\\0 & 1 & d\\0 & 0 & 1\end{pmatrix} \times \begin{pmatrix}1 & 0 & a\\0 & 1 & b\\0 & 0 & 1\end{pmatrix} = \begin{pmatrix}1 & 0 & a+c\\0 & 1 & b+d\\0 & 0 & 1\end{pmatrix}$$

$\Rightarrow T_1 \times T_2 = T_2 \times T_1 \Rightarrow$ does commute

For $S_1 \times S_2$

$$\begin{pmatrix}a & 0 & 0\\0 & b & 0\\0 & 0 & 1\end{pmatrix} \times \begin{pmatrix}c & 0 & 0\\0 & d & 0\\0 & 0 & 1\end{pmatrix} = \begin{pmatrix}a \times c & 0 & 0\\b \times d & 0 & 0\\0 & 0 & 1\end{pmatrix}$$

For $S_2 \times S_1$

$$\begin{pmatrix}c & 0 & 0\\0 & d & 0\\0 & 0 & 1\end{pmatrix} \times \begin{pmatrix}a & 0 & 0\\0 & b & 0\\0 & 0 & 1\end{pmatrix} = \begin{pmatrix}a \times c & 0 & 0\\b \times d & 0 & 0\\0 & 0 & 1\end{pmatrix}$$

$\Rightarrow S_1 \times S_2 = S_2 \times S_1 \Rightarrow$ does commute

For $R \times T$

$$\begin{pmatrix}a & b & 0\\c & d & 0\\0 & 0 & 1\end{pmatrix} \times \begin{pmatrix}1 & 0 & e\\0 & 1 & f\\0 & 0 & 1\end{pmatrix} = \begin{pmatrix}a & b & a \times e + b \times f\\c & d & c \times e + d \times f\\0 & 0 & 1\end{pmatrix}$$

For $T \times R$

$$\begin{pmatrix}1 & 0 & e\\0 & 1 & f\\0 & 0 & 1\end{pmatrix} \times \begin{pmatrix}a & b & 0\\c & d & 0\\0 & 0 & 1\end{pmatrix} = \begin{pmatrix}a & b & e\\c & d & f\\0 & 0 & 1\end{pmatrix}$$

$\Rightarrow R \times T \neq T \times R \Rightarrow$ does not commute

For $R \times S$

$$\begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} e & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a \times e & b \times f & 0 \\ c \times f & d \times f & 0 \times e + d \times f \\ 0 & 0 & 1 \end{pmatrix}$$

For $S \times R$

$$\begin{pmatrix} e & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a \times e & b \times f & 0 \\ c \times f & d \times f & 0 \times e + d \times f \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow R \times S = S \times R \Rightarrow$ does commute

For $S \times T$

$$\begin{pmatrix} 1 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} c & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c & 0 & a \\ 0 & d & b \\ 0 & 0 & 1 \end{pmatrix}$$

For $T \times S$

$$\begin{pmatrix} c & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c & 0 & a \times c \\ d & 0 & b \times d \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow S \times T \neq T \times S \Rightarrow$ does not commute

1.1.2

a)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \\ z_1 & z_2 & \cdots & z_n \end{pmatrix} = \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ z_1 & z_2 & \cdots & z_n \\ y_1 & y_2 & \cdots & y_n \end{pmatrix}$$

b)

$$\begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \\ z_1 & z_2 & \cdots & z_n \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} \sum\limits_{i=1} x_i \\ \sum\limits_{i=1} y_i \\ \sum\limits_{i=1} z_i \end{pmatrix}$$

c)

$$L(p) = length(p_x - ((p_x \cdot (\vec{e} + \vec{d} \cdot t)) \cdot \vec{d} \cdot t) - \vec{e})$$
$$Vector = (L(p_x), L(p_{x+1}), ..., L(p_{x+n})) \qquad n \widehat{=} Anz.Punkte$$

Berechnung für:
$p_1 = (-1, -1, 1); p_2 = (2, 1, -2); p_3 = (2, 1, -3); p_4 = (-1, -2, 1); p_5 = (3, -1, 0)$

$$L(p_1) = length(p_1 - ((p_1 \cdot (\vec{e} + \vec{d} \cdot t)) \cdot \vec{d} \cdot t) - \vec{e})$$
$$L(p_2) = length(p_2 - ((p_2 \cdot (\vec{e} + \vec{d} \cdot t)) \cdot \vec{d} \cdot t) - \vec{e})$$
$$L(p_3) = length(p_3 - ((p_3 \cdot (\vec{e} + \vec{d} \cdot t)) \cdot \vec{d} \cdot t) - \vec{e})$$
$$L(p_4) = length(p_4 - ((p_4 \cdot (\vec{e} + \vec{d} \cdot t)) \cdot \vec{d} \cdot t) - \vec{e})$$
$$L(p_5) = length(p_5 - ((p_5 \cdot (\vec{e} + \vec{d} \cdot t)) \cdot \vec{d} \cdot t) - \vec{e})$$

$$Vector = (L(p_1), L(p_2), L(p_3), L(p_4), L(p_5))$$

$$M = \begin{pmatrix} M_{11} & M_{12} & M_{13} & t_x \\ M_{21} & M_{22} & M_{23} & t_y \\ M_{31} & M_{32} & M_{33} & t_z \\ p_x & p_y & p_z & 1 \\ \end{pmatrix}; e = \begin{pmatrix} 5 \\ 10 \\ 5 \end{pmatrix} = t; p = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$z_\phi = \frac{\begin{pmatrix}0\\0\\1\end{pmatrix} \times p_x}{1 \times |p_x|} \Rightarrow \begin{pmatrix}cos(z_\phi) & -sin(z_\phi) & 0\\ sin(z_\phi) & cos(z_\phi) & 0\\ 0 & 0 & 1\end{pmatrix} = R_z$$

$$y_\phi = \frac{\begin{pmatrix}0\\1\\0\end{pmatrix} \times p_x}{1 \times |p_x|} \Rightarrow \begin{pmatrix}cos(y_\phi) & 0 & sin(y_\phi)\\ 0 & 1 & 0\\ -sin(z_\phi) & 0 & cos(y_\phi)\end{pmatrix} = R_y$$

$$x_\phi = \frac{\begin{pmatrix}1\\0\\0\end{pmatrix} \times p_x}{1 \times |p_x|} \Rightarrow \begin{pmatrix}0 & 0 & 0\\ 0 & cos(x_\phi) & -sin(x_\phi)\\ 0 & sin(x_\phi) & cos(x_\phi)\end{pmatrix} = R_x$$

$$M = R_x \times R_y \times R_z$$

$$\Rightarrow Vector = \begin{pmatrix}6.323746006808568\\ 2.672777625063053\\ 1.6528549605644152\\ 6.432049954608068\\ 4.731031538265404\end{pmatrix}$$

$$\Rightarrow z_{far} = p_4 = 6.432049954608068; z_{near} = p_3 = 1.6528549605644152$$