Computer Graphics - Excercise 1

1.1.1

a)

- Matrix Multiplication $\widehat{=}$ transformation
- Inverse of a transformation $\tilde{T}^{-1}$

$$
\begin{aligned}
\Rightarrow \qquad & \tilde{T} * \tilde{T}^{-1} = \tilde{T}^{-1} * \tilde{T} = \tilde{E} && \mid *\tilde{p} \\
& \tilde{T}^{-1} * \tilde{T} * \tilde{p} = \tilde{E} * \tilde{p} && \Big| \, \tilde{T} * \tilde{p} = \tilde{p'} \\
\Rightarrow \qquad & \tilde{E} * \tilde{p} = \tilde{p}, \quad \tilde{T}^{-1} * \tilde{p'} = \tilde{p}
\end{aligned}
$$

b)

For $R_1 \times R_2$

$$
\begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} e & f & 0 \\ g & h & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a \times e + b \times g & a \times f + b \times h & 0 \\ c \times e + d \times g & c \times f + d \times h & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

For $R_2 \times R_1$

$$
\begin{pmatrix} e & f & 0 \\ g & h & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a \times e + c \times f & b \times e + d \times f & 0 \\ a \times g + c \times h & b \times g + d \times h & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

$\Rightarrow R_1 \times R_2 \neq R_2 \times R_1 \Rightarrow$ does not commute

For $T_1 \times T_2$

$$\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & c \\ 0 & 1 & d \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & a+c \\ 0 & 1 & b+d \\ 0 & 0 & 1 \end{pmatrix}$$

For $T_2 \times T_1$

$$\begin{pmatrix} 1 & 0 & c \\ 0 & 1 & d \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & a+c \\ 0 & 1 & b+d \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow T_1 \times T_2 = T_2 \times T_1 \Rightarrow$ does commute

For $S_1 \times S_2$

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} c & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a \times c & 0 & 0 \\ b \times d & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

For $S_2 \times S_1$

$$\begin{pmatrix} c & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a \times c & 0 & 0 \\ b \times d & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow S_1 \times S_2 = S_2 \times S_1 \Rightarrow$ does commute

For $R \times T$

$$\begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & e \\ 0 & 1 & f \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a & b & a \times e + b \times f \\ c & d & c \times e + d \times f \\ 0 & 0 & 1 \end{pmatrix}$$

For $T \times R$

$$\begin{pmatrix} 1 & 0 & e \\ 0 & 1 & f \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow R \times T \neq T \times R \Rightarrow$ does not commute

For $R \times S$

$$\begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} e & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a \times e & b \times f & 0 \\ c \times f & d \times f & 0 \times e + d \times f \\ 0 & 0 & 1 \end{pmatrix}$$

For $S \times R$

$$\begin{pmatrix} e & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a \times e & b \times f & 0 \\ c \times f & d \times f & 0 \times e + d \times f \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow R \times S = S \times R \Rightarrow$ does commute

For $S \times T$

$$\begin{pmatrix} 1 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} c & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c & 0 & a \\ 0 & d & b \\ 0 & 0 & 1 \end{pmatrix}$$

For $T \times S$

$$\begin{pmatrix} c & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c & 0 & a \times c \\ d & 0 & b \times d \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow S \times T \neq T \times S \Rightarrow$ does not commute

1.1.2

a)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \\ z_1 & z_2 & \cdots & z_n \end{pmatrix} = \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ z_1 & z_2 & \cdots & z_n \\ y_1 & y_2 & \cdots & y_n \end{pmatrix}$$

b)

$$\begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \\ z_1 & z_2 & \cdots & z_n \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} \sum\limits_{i=1} x_i \\ \sum\limits_{i=1} y_i \\ \sum\limits_{i=1} z_i \end{pmatrix}$$

c)

$$
\begin{aligned}
L(p) &= length(p_x - ((p_x \cdot (\vec{e} + \vec{d} * t)) * \vec{d} * t) - \vec{e}) \\
Vector &= (L(p_x), L(p_{x+1}), ..., L(p_{x+n})) \qquad\qquad n \hat{=} Anz.Punkte
\end{aligned}
$$

Berechnung: