

Computer Graphics - Exercise 2

$$f(u, v) = R \begin{pmatrix} \cos(u) \\ \sin(u) \\ 0 \end{pmatrix} + r \begin{pmatrix} \cos(u) \times \cos(v) \\ \sin(u) \times \cos(v) \\ \sin(v) \end{pmatrix}$$

2.1

a)

$$\frac{\delta f}{\delta u}(u, v) = \begin{pmatrix} R(-\sin(u)) + r(\cos(v) \times -\sin(u)) \\ R \times \cos(u) + r(\cos(v) \times \cos(u)) \\ 0 \end{pmatrix}$$

$$\frac{\delta f}{\delta v}(u, v) = \begin{pmatrix} r(\cos(u) \times (-\sin(v))) \\ r(-\sin(v) \times \sin(u)) \\ r \times \cos(v) \end{pmatrix}$$

\Rightarrow Jacobian :

$$J_f(u, v) = \begin{pmatrix} R \times (-\sin(u)) + r(\cos(v) \times -\sin(u)) & r(\cos(u) \times (-\sin(v))) \\ \sin(u) \times \cos(v) & r(-\sin(v) \times \sin(u)) \\ 0 & r \times \cos(v) \end{pmatrix}$$

b)

$$I_\tau^s = \begin{pmatrix} E & F \\ F & G \end{pmatrix} \text{ mit } s_u = \frac{\delta f}{\delta u}(u, v), s_v = \frac{\delta f}{\delta v}(u, v)$$

$$E = \|s_u\|^2 = (r(\cos(u)) + R)^2$$

$$F = \langle s_u, s_v \rangle = 0$$

$$G = \|s_v\|^2 = r^2$$

$$\Rightarrow I_\tau^f = \begin{pmatrix} (r(\cos(u)) + R)^2 & 0 \\ 0 & r^2 \end{pmatrix}$$

c)

Aus der Vorlesung: $dA = \sqrt{\det I_\tau^f} du dv$

$$\begin{aligned}\Rightarrow dA &= \sqrt{\det \begin{pmatrix} (r(\cos(u)) + R)^2 & 0 \\ 0 & r^2 \end{pmatrix}} du dv \\ \Rightarrow dA &= r(r(\cos(u)) + R) du dv\end{aligned}$$

$$\Rightarrow A = \int_0^{2\pi} \int_0^{2\pi} r(r(\cos(u)) + R) du dv = 2\pi r \times \int_0^{2\pi} r(\cos(u)) + R dv =$$

$$2\pi r [Rv + r(\sin(v))]_0^{2\pi} = 2\pi r \times (2\pi R) = \underline{\underline{4\pi^2 Rr}} \text{ t}$$