

## Computer Graphics - Exercise 2

$$f(u, v) = R \begin{pmatrix} \cos(u) \\ \sin(u) \\ 0 \end{pmatrix} + r \begin{pmatrix} \cos(u) \times \cos(v) \\ \sin(u) \times \cos(v) \\ \sin(v) \end{pmatrix}$$

### 2.1

a)

$$\frac{\delta f}{\delta u}(u, v) = \begin{pmatrix} R(-\sin(u)) + r(\cos(v) \times -\sin(u)) \\ R \times \cos(u) + r(\cos(v) \times \cos(u)) \\ 0 \end{pmatrix}$$

$$\frac{\delta f}{\delta v}(u, v) = \begin{pmatrix} r(\cos(u) \times (-\sin(v))) \\ r(-\sin(v) \times \sin(u)) \\ r \times \cos(v) \end{pmatrix}$$

$\Rightarrow$  Jacobian :

$$J_f(u, f) = \begin{pmatrix} R \times (-\sin(u)) + r(\cos(v) \times -\sin(u)) & r(\cos(u) \times (-\sin(v))) \\ \sin(u) \times \cos(v) & r(-\sin(v) \times \sin(u)) \\ 0 & r \times \cos(v) \end{pmatrix}$$

b)

$$I_\tau^s = \begin{pmatrix} E & F \\ F & G \end{pmatrix} \text{ mit } s_u = \frac{\delta f}{\delta u}(u, v), s_v = \frac{\delta f}{\delta v}(u, v)$$

$$E = \|s_u\|^2 =$$

$$\begin{aligned} & \sqrt{(R(-\sin(u)) + r(\cos(v) \times -\sin(u)))^2 + (R \times \cos(u) + r(\cos(v) \times \cos(u)))^2 + 0^2} = \\ & \sqrt{R^2((-\sin(u))^2 + \cos(u)^2) + 2Rrcos(v)((-\sin(u))^2 + \cos(u)^2) + r^2\cos(v)((-\sin(u))^2 + \cos(u)^2)} = \\ & \sqrt{(R^2 + 2Rrcos(v) + r^2\cos(v))^2} = (r(\cos(u)) + R)^2 \end{aligned}$$

$$F = \langle s_u, s_v \rangle = 0$$

$$G = \|s_v\|^2 =$$

$$\begin{aligned} & \sqrt{(r(\cos(u) \times (-\sin(v))))^2 + (r(-\sin(v) \times \sin(u)))^2 + (r \times \cos(v))^2} = \\ & \sqrt{(r^2 \times \cos(u)^2 \times (-\sin(v))^2) + (r^2((-\sin(v))^2 \times \sin(u)^2)) + (r^2 \times \cos(v)^2)} = \\ & \sqrt{r^2(\sin(v)^2(\cos(u)^2 \times \sin(u)^2) + \cos(v)^2)} = \\ & \sqrt{r^2(\sin(v)^2 + \cos(v)^2)} = r^2 \end{aligned}$$

$$\Rightarrow I_\tau^f = \begin{pmatrix} (r(\cos(u)) + R)^2 & 0 \\ 0 & r^2 \end{pmatrix}$$

c)

Aus der Vorlesung:  $dA = \sqrt{\det I_r^f} du dv$

$$\begin{aligned}\Rightarrow dA &= \sqrt{\det \begin{pmatrix} (r(\cos(u)) + R)^2 & 0 \\ 0 & r^2 \end{pmatrix}} du dv \\ \Rightarrow dA &= r(r(\cos(u)) + R) du dv\end{aligned}$$

$$\begin{aligned}\Rightarrow A &= \int_0^{2\pi} \int_0^{2\pi} r(r(\cos(u)) + R) du dv = 2\pi r \times \int_0^{2\pi} r(\cos(u)) + R dv = \\ &2\pi r [Rv + r(\sin(v))]_0^{2\pi} = 2\pi r \times (2\pi R) = \underline{\underline{4\pi^2 Rr}}\end{aligned}$$