Computer Graphics - Excercise 2

3.1.1

(a)

The more obtuse the triangle, the more acute the angle of the corner of the voronoi area gets. Because the both sides of this corner have to be in a 90° angle to the sides of the traingle.

(b)

The formula for calculating the surface of a triangle is: $\frac{1}{2} * h * b$ As for the red triangle: $\frac{1}{2} * h * b$ We define: $h = \frac{||\underline{p}_i - \underline{p}_j||}{2}$ and $b = b_1 + b_2$ where b_1 is left part of the baseline facing α_{ij} and b_2 is the right one facing β_{ij}. In addition we define the vertex of the traingle facing α_{ij} to be A and the vertex of the triangle facing β_{ij} to be B. The distance between p_i and A (R) is equal to $\frac{||\underline{p}_i - \underline{p}_j||}{2sin(\alpha_{ij}}$. It can be assumed that the angle $\underline{p}_S \underline{A} \underline{p}_i = \alpha_{ij}$ with p_S as the vertex where the red triangle crosses the edge from $\underline{p}_i$ to $\underline{p}_j$. As a result it can be assumed that $cos(\alpha_{ij}) = \frac{b_1}{R}$ and also $b_1 = cos(\alpha_{ij}) \times R$
As a result the surface of the left part of the red triangle can be calculated:

$$
\begin{aligned}
A_\alpha(i,j) &= \frac{1}{2} \times b_1 \times \frac{||\underline{p}_i - \underline{p}_j||}{2} \\
A_\alpha(i,j) &= \frac{1}{4} \times cos(\alpha_{ij}) \times R \times ||\underline{p}_i - \underline{p}_j|| \\
A_\alpha(i,j) &= \frac{1}{4} \times \frac{cos(\alpha_{ij})}{2 \times sin(\alpha_{ij})} \times R \times ||\underline{p}_i - \underline{p}_j||^2 \\
A_\alpha(i,j) &= \frac{1}{8} \times cot(\alpha_{ij}) \times R \times ||\underline{p}_i - \underline{p}_j||^2
\end{aligned}
$$

The same goes for the right side and as a result if we combine both:

$$
\begin{aligned}
A(i,j) &= \frac{1}{8} \times cot(\alpha_{ij}) \times R \times ||\underline{p}_i - \underline{p}_j||^2 + \frac{1}{8} \times cot(\beta_{ij}) \times R \times ||\underline{p}_i - \underline{p}_j||^2 \\
A(i,j) &= \frac{1}{8} \times (cot(\alpha_{ij}) + cot(\beta_{ij})) \times R \times ||\underline{p}_i - \underline{p}_j||^2
\end{aligned}
$$

If we sum up all the triangles T with the index j of the voronoi-area this is the result:

$$
\begin{aligned}
A_{voronoi-area}(i) &= \sum_{j \in T(i)} \frac{1}{8} \times (cot(\alpha_{ij}) + cot(\beta_{ij})) \times R \times ||\underline{p}_i - \underline{p}_j||^2 \\
A_{voronoi-area}(i) &= \frac{1}{8} \times \sum_{j \in T(i)} (cot(\alpha_{ij}) + cot(\beta_{ij})) \times R \times ||\underline{p}_i - \underline{p}_j||^2
\end{aligned}
$$

3.1.2

(a)

We know every vertex(v) has three edges(e) and 2 faces(f). In addition every edge(e) can separated into two half edges (e_h), this means $e_h = 2 * e = 6 * v$.
If we add the memory for every part it results in a formula:

$$\text{memory} = v * 16\text{bytes} + e * 4\text{bytes} + e_h * 16\text{bytes} + f * 4\text{bytes}$$
$$\rightarrow \text{through assumptions:}$$
$$\text{memory} = v * (16 + 3 * 4 + 6 * 16 + 2 * 4)\text{bytes} = v * 132\text{bytes}$$

(b)

Because in a quad mash two triangles are combined into one quad, the resulting faces will be reduced by a half. The resulting ratio will be 1:3:1 (for v : e : f)

(c)

We know every vertex(v) has three edges(e) and one face(f). In addition every edge(e) can separated into two half edges (e_h), this means $e_h = 2 * e = 6 * v$.
If we add the memory for every part it results in a formula:

$$\text{memory} = v * 16\text{bytes} + e * 4\text{bytes} + e_h * 16\text{bytes} + f * 4\text{bytes}$$
$$\rightarrow \text{through assumptions:}$$
$$\text{memory} = v * (16 + 3 * 4 + 6 * 16 + 1 * 4)\text{bytes} = v * 132\text{bytes}$$