Computer Graphics - Excercise 2

$$f(u,v) = R\begin{pmatrix} cos(u) \\ sin(u) \\ 0 \end{pmatrix} + r\begin{pmatrix} cos(u) \times cos(v) \\ sin(u) \times cos(v) \\ sin(v) \end{pmatrix}$$

2.1

a)

$$\frac{\delta f}{\delta u}(u,v) = \begin{pmatrix} R(-sin(u)) + r(cos(v) \times -sin(u)) \\ R \times cos(u) + r(cos(v) \times cos(u)) \\ 0 \end{pmatrix}$$

$$\frac{\delta f}{\delta v}(u,v) = \begin{pmatrix} r(cos(u) \times (-sin(v))) \\ r(-sin(v) \times sin(u)) \\ r \times cos(v) \end{pmatrix}$$

$$\Rightarrow Jacobian:$$

$$J_f(u,f) = \begin{pmatrix} R \times (-sin(u)) + r(cos(v) \times -sin(u)) & r(cos(u) \times (-sin(v))) \\ sin(u) \times cos(v) & r(-sin(v) \times sin(u)) \\ 0 & r \times cos(v) \end{pmatrix}$$

b)

$I_\tau^s = \begin{pmatrix} E & F \\ F & G \end{pmatrix}$ mit $s_u = \frac{\delta f}{\delta u}(u,v), s_v = \frac{\delta f}{\delta v}(u,v)$

$E = ||s_u||^2 =$
$\sqrt{(R(-sin(u)) + r(cos(v) \times -sin(u)))^2 + (R \times cos(u) + r(cos(v) \times cos(u)))^2 + 0^2}^2 =$
$\sqrt{R^2((-sin(u))^2 + cos(u)^2) + 2Rrcos(v)((-sin(u))^2 + cos(u)^2) + r^2cos(v)((-sin(u))^2 + cos(u)^2)}^2 =$
$\sqrt{(R^2 + 2Rrcos(v) + r^2cos(v)^2}^2 = (r(cos(u)) + R)^2$

$F =< s_u, s_v >= 0$

$G = ||s_v||^2 =$
$\sqrt{(r(cos(u) \times (-sin(v))))^2 + (r(-sin(v) \times sin(u)))^2 + (r \times cos(v))^2}^2 =$
$\sqrt{(r^2 \times cos(u)^2 \times (-sin(v))^2) + (r^2((-sin(v))^2 \times sin(u)^2)) + (r^2 \times cos(v)^2)}^2 =$
$\sqrt{r^2(sin(v)^2(cos(u)^2 \times sin(u)^2) + cos(v)^2}^2 =$
$\sqrt{r^2(sin(v)^2 + cos(v)^2}^2 = r^2$

$\Rightarrow I_\tau^f = \begin{pmatrix} (r(cos(u)) + R)^2 & 0 \\ 0 & r^2 \end{pmatrix}$

c)

Aus der Vorlesung: $dA = \sqrt{det I_\tau^f}\, du\, dv$

$$\Rightarrow dA = \sqrt{det \begin{pmatrix} (r(cos(u)) + R)^2 & 0 \\ 0 & r^2 \end{pmatrix}}\, du\, dv$$
$$\Rightarrow dA = r(r(cos(u)) + R)\, du\, dv$$

$\Rightarrow A = \int_0^{2\pi} \int_0^{2\pi} r(r(cos(u)) + R)\, du\, dv = 2\pi r \times \int_0^{2\pi} r(cos(u)) + R\, dv =$

$2\pi r[Rv + r(sin(v)]_0^{2\pi} = 2\pi r \times (2\pi R) = \underline{\underline{4\pi^2 Rr}}$