

Computer Graphics - Exercise 1

1.1.1

a)

- Matrix Multiplication $\hat{=}$ transformation
- Inverse of a transformation \tilde{T}^{-1}

$$\begin{aligned} \Rightarrow \quad & \tilde{T} * \tilde{T}^{-1} = \tilde{T}^{-1} * \tilde{T} = \tilde{E} & \left| \begin{array}{l} * \tilde{p} \\ \tilde{T} * \tilde{p} = \tilde{p}' \end{array} \right. \\ & \tilde{T}^{-1} * \tilde{T} * \tilde{p} = \tilde{E} * \tilde{p} \\ \Rightarrow \quad & \tilde{E} * \tilde{p} = \tilde{p}, \quad \tilde{T}^{-1} * \tilde{p}' = \tilde{p} \end{aligned}$$

b)

For $R_1 \times R_2$

$$\begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} e & f & 0 \\ g & h & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a \times e + b \times g & a \times f + b \times h & 0 \\ c \times e + d \times g & c \times f + d \times h & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

For $R_2 \times R_1$

$$\begin{pmatrix} e & f & 0 \\ g & h & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a \times e + c \times f & b \times e + d \times f & 0 \\ a \times g + c \times h & b \times g + d \times h & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow R_1 \times R_2 \neq R_2 \times R_1 \Rightarrow$ does not commute

For $T_1 \times T_2$

$$\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & c \\ 0 & 1 & d \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & a+c \\ 0 & 1 & b+d \\ 0 & 0 & 1 \end{pmatrix}$$

For $T_2 \times T_1$

$$\begin{pmatrix} 1 & 0 & c \\ 0 & 1 & d \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & a+c \\ 0 & 1 & b+d \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow T_1 \times T_2 = T_2 \times T_1 \Rightarrow$ does commute

For $S_1 \times S_2$

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} c & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a \times c & 0 & 0 \\ b \times d & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

For $S_2 \times S_1$

$$\begin{pmatrix} c & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a \times c & 0 & 0 \\ b \times d & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow S_1 \times S_2 = S_2 \times S_1 \Rightarrow$ does commute

For $R \times T$

$$\begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & e \\ 0 & 1 & f \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a & b & a \times e + b \times f \\ c & d & c \times e + d \times f \\ 0 & 0 & 1 \end{pmatrix}$$

For $T \times R$

$$\begin{pmatrix} 1 & 0 & e \\ 0 & 1 & f \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow R \times T \neq T \times R \Rightarrow$ does not commute

For $R \times S$

$$\begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} e & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a \times e & b \times f & 0 \\ c \times f & d \times f & 0 \times e + d \times f \\ 0 & 0 & 1 \end{pmatrix}$$

For $S \times R$

$$\begin{pmatrix} e & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a \times e & b \times f & 0 \\ c \times f & d \times f & 0 \times e + d \times f \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow R \times S = S \times R \Rightarrow$ does commute

For $S \times T$

$$\begin{pmatrix} 1 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} c & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c & 0 & a \\ 0 & d & b \\ 0 & 0 & 1 \end{pmatrix}$$

For $T \times S$

$$\begin{pmatrix} c & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c & 0 & a \times c \\ d & 0 & b \times d \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow S \times T \neq T \times S \Rightarrow$ does not commute

1.1.2

a)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \\ z_1 & z_2 & \cdots & z_n \end{pmatrix} = \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ z_1 & z_2 & \cdots & z_n \\ y_1 & y_2 & \cdots & y_n \end{pmatrix}$$

b)

$$\begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \\ z_1 & z_2 & \cdots & z_n \end{pmatrix} \times n \downarrow \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} \sum_{i=1} x_i \\ \sum_{i=1} y_i \\ \sum_{i=1} z_i \end{pmatrix}$$

c)

$$\tilde{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & t_x \\ M_{21} & M_{22} & M_{23} & t_y \\ M_{31} & M_{32} & M_{33} & t_z \\ p_x & p_y & p_z & 1 \end{pmatrix}; e = \begin{pmatrix} 5 \\ 10 \\ 5 \end{pmatrix} = t; p = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$z_\phi = \frac{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times p_x}{1 \times |p_x|} \Rightarrow \begin{pmatrix} \cos(z_\phi) & -\sin(z_\phi) & 0 \\ \sin(z_\phi) & \cos(z_\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix} = R_z$$

$$y_\phi = \frac{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times p_x}{1 \times |p_x|} \Rightarrow \begin{pmatrix} \cos(y_\phi) & 0 & \sin(y_\phi) \\ 0 & 1 & 0 \\ -\sin(z_\phi) & 0 & \cos(y_\phi) \end{pmatrix} = R_y$$

$$x_\phi = \frac{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times p_x}{1 \times |p_x|} \Rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & \cos(x_\phi) & -\sin(x_\phi) \\ 0 & \sin(x_\phi) & \cos(x_\phi) \end{pmatrix} = R_x$$

$$M = R_x \times R_y \times R_z$$

Berechnung für:

$$p_1 = (-1, -1, 1); p_2 = (2, 1, -2); p_3 = (2, 1, -3); p_4 = (-1, -2, 1); p_5 = (3, -1, 0)$$

$$\Rightarrow Vector = \begin{pmatrix} p_1 \times \tilde{M}_1 \\ p_2 \times \tilde{M}_2 \\ p_3 \times \tilde{M}_3 \\ p_4 \times \tilde{M}_4 \\ p_5 \times \tilde{M}_5 \end{pmatrix} = \begin{pmatrix} 6.323746006808568 \\ 2.672777625063053 \\ 1.6528549605644152 \\ 6.432049954608068 \\ 4.731031538265404 \end{pmatrix}$$

$$\Rightarrow z_{far} = p_4 = 6.432049954608068; z_{near} = p_3 = 1.6528549605644152$$